Reply to "the Comment on 'Gauge Invariance and k_T-Factorization of Exclusive Processes'"

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Abstract

A new method is proposed to calculate wave functions in k_T-factorization in [1] as a comment about our paper [2]. We point out that the results obtained with the method are in conflict with the translation invariance and depend on the chosen contours for loop-integrals. Therefore, the method is in principle unacceptable and the results with the method cannot be correct.

Recently we have pointed out in [2] that the k_T-factorization performed with off-shell partons is in general gauge variant and violated, because the perturbative coefficient function H contains gauge-dependent light-cone singularities and gauge-dependent μ-dependence. The k_T-factorization for the transition $\pi^0 + \gamma^* \to \gamma$ has been examined in [3] in Feynman gauge at one-loop level. It is claimed in [3] that the perturbative coefficient function H calculated with off-shell partons is gauge-invariant. A proof of the gauge invariance at any-loop level is given in [3], where one shows the gauge invariance at $N+1$-loop level with the induction method by assuming the gauge invariance at N-loop level. But, the gauge invariance of the one-loop $H^{(1)}$ is not shown. This motivates us to study the problem in [2].

To determine the perturbative coefficient function H in the k_T-factorization one replaces π^0 with an off-shell quark pair. With the quark pair one calculates the form factor and the wave function. H receives contributions from the form factor and the wave function. In [2] it is shown with covariance arguments that the form factor does not contain the gauge-dependent light-cone singularities. The singularities come from wave functions. In the updated comment made by Li and Mishima[1] a new method is proposed to calculate wave functions with off-shell partons. It is found that the gauge-dependent light-cone singularities disappear. In this updated reply we point out unacceptable flaws of the method.

The definition of the wave function for π^0 moving in the z-direction is [2, 3]:

$$\phi(x, k_T, \zeta, \mu) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ixP^+ z^- - i\vec{z}_\perp \cdot \vec{k}_T} \langle 0|\bar{q}(0) L_u^\dagger(\infty, 0)\gamma^+\gamma_5 L_u(\infty, z) q(z)|\pi^0(P)\rangle|_{z^+=0}. \quad (1)$$

We have taken the light-cone coordinate system in which π^0 has the large momentum component P^+. L_u is a gauge link along the direction $u^\mu = (u^+, u^-, 0, 0)$ with $u^- \ll u^+$ and ζ is defined as $\zeta^2 \approx 4(u \cdot P)^2/u^2$. In the definition there is the momentum conservation in the +- and transverse direction due to the translation invariance. Because of this and $z^+ = 0$ all $-$-components of momenta in loops are integrated over from $-\infty$ to ∞. It is noted that ϕ only depends on x, k_T, ζ and μ as indicated in the above.

To extract the one-loop coefficient $H^{(1)}$ one needs to calculate ϕ of an off-shell parton pair instead of π^0 at one-loop, as required in the k_T-factorization[3]. Then one calculates the convolution of

ϕ with $H^{(0)}$, i.e., $\phi^{(1)} \otimes H^{(0)}$ as a part of contributions to $H^{(1)}$. The mentioned singularity in ϕ comes from Fig.2a, 2b and 2c in [2, 1], where one has an integral of q^- from $-\infty$ to ∞. q^- is the component of the momentum q carried by the exchanged gluon in the figures, the component q^+ and $q_\perp$ are fixed in ϕ by the momentum conservation. The integral can be performed by a standard contour integral in the q^- complex-plane. One can takes a closed contour like those indicated in Fig.5 of [1] for $-R \leq q^- \leq R$ at first step by adding semicircles. After the integration one takes the limit $R \to \infty$. In this limit one finds that all contributions to ϕ from semicircles in Fig.5 of [1] are zero. The remaining contributions to ϕ are only from poles in the complex plane and contain the light-cone singularity[2, 1]. In the limit the contribution ϕ_{2b} from Fig.2b in [2, 1] to ϕ is nonzero only for $0 < q^+ < k_1^+$ with $k_1^\mu = (k_1^+, 0, \vec{k}_{1\perp})$ as the momentum carried by the off-shell quark.

In [1] it is suggested that one should keep these contributions to ϕ from the semicircles to calculate the convolution $\phi^{(1)} \otimes H^{(0)}$, which contains integrations over x and k_T. x is related to q^+ through $xP^+ = k_1^+ - q^+$ in [2, 1]. After finishing the integrations in the convolution one then takes the limit $R \to \infty$. In this way the light-cone singularity disappears because the contributions from semicircles given in Fig.5 [1] give nonzero contributions to the convolution in the limit. Since the limit is taken after the convolution, this implies that ϕ depends on an extra parameter R which is in fact a cut-off for the large $|q^-|$. The existence of a finite R actually is in conflict with the translational invariance. We take the contribution ϕ_{2b} as an example to illustrate this in more detail in the following.

It is well-known that the wave function in Eq.(1) is zero for $x < 0$ and $x > 1$. The boundary $x = 1$ can be obtained by inserting a complete sum of intermediate states in Eq.(1) and using the translational invariance. With the finite R ϕ_{2b} is not zero for $q^+ < 0$. This nonzero contribution is crucial to cancel the light-cone singularity of ϕ_{2b} with $q^+ > 0$[1]. If one takes the limit $R \to \infty$ before the convolution, ϕ_{2b} is zero for $q^+ < 0$. From the translation invariance one can show that ϕ_{2b} or contributions from similar class of diagrams are zero for $q^+ < 0$ without any calculation. The contributions from Fig.2b in [2, 1] and its higher-order corrections, in which there is no gluon attached to the antiquark line and the gauge link $L_u(\infty, z)$, are actually proportional to the contributions to a distribution of a single quark with the momentum k_1. The distribution can be defined by replacing π^0 in Eq.(1) with the single off-shell quark and delete some operators in Eq.(1) as in the following:

$$\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{iyk_1^+ z^- - i\vec{z}_\perp \cdot \vec{k}_T} \langle 0| \left(L_u^\dagger(\infty, 0) - 1 \right) q(z) |q(k_1)\rangle|_{z^+=0}. \tag{2}$$

ϕ_{2b} is proportional to one of the leading order contributions of the above distribution with $xP^+ = yk_1^+$. Inserting a complete sum of intermediate states and using the translation invariance, we have that the above distribution must be zero for $y > 1$. Here y is related to q^+ through $yk_1^+ = k_1^+ - q^+$. One can conclude without any calculation that the above distribution and ϕ_{2b} must be zero for $q^+ < 0$, or the contributions from similar diagrams are zero if the out going quark line carries a $+$-momentum component is larger than that of the incoming quark line. This conclusion is in agreement with the physical expectation that partons entering hard scattering have positive $+$-energy. Therefore, $\phi_{2b} \neq 0$ with $q^+ < 0$ is in conflict with the translation invariance. To avoid the conflict one has to take the limit $R \to \infty$ before the convolution. This results in $\phi_{2b} = 0$ for $q^+ < 0$.

In fact there is certain ambiguity in ϕ_{2b} if one does not take the limit $R \to \infty$. In the method, one actually changes the order of the integrations in the convolution $\phi_{2b} \otimes H^{(0)}$ with the q^--integration in ϕ_{2b} by implementing certain prescriptions, where one first deforms the contours along the real axis in the q^--plane for the q^--integral in ϕ to those in Fig.5 of [1]. With these contours one finds

the nonexistence of the light-cone singularity. If one flips the closed contour in Fig.5b of [1], i.e., by changing the contour in the upper-half q^--plane into the lower-half q^--plane, one will find a different result from Fig.5b than that given in Eq.(12) of [1] by noting that fact: The contribution from the single pole in the upper-half q^--plane is the same as that from the double pole in the lower-half q^--plane, and with the flipped contour the corresponding integral of θ from $\pi \to 0$ in Eq.(12) of [1] is changed to that of θ from $-\pi \to 0$. The final results with the flipped contour will still contain the light-cone singularity. The cancelation of the light-cone singularity only happens with certain contours like those given in Fig.5 of [1]. This is in conflict with the general expectation that physical results should not depend contours used for loop-integrals.

It is interesting to note that one will not find the singularity in the convolution if one simply changes the order of the integrations of $\phi_{2b} \otimes H^{(0)}$ by ignoring the constraint of allowed q^+-regions from the translational invariance, where one first integrates over q^+ by taking the region with $q^+ < 0$ into account and then perform the integration over q^-. However, from the translational invariance we know that the region with $q^+ < 0$ should be excluded in the integration of q^+. Taking this into account, ϕ_{2b} and its convolution with $H^{(0)}$ has the mentioned light-cone singularity.

The above discussion tells that the final results with the method will depend on chosen contours. The method implies a modification for the wave function of parton states through implementing a cut-off and specially chosen contours. It is unclear how to implement the modification to the wave function at hadron level, i.e., to that in Eq.(1). With the implement the calculated wave function is in fact no longer the wave function employed in the k_T-factorization in Eq.(1). Since the results obtained with the method are in conflict with the translational invariance and depend on the chosen contours, the proposed method is in principle unacceptable.

To conclude: In the suggested method of [1] an extra cut-off is introduced into wave functions and special contours for $-$-components of loop momenta are taken. The results obtained with the method are in conflict with the translation invariance and depend on the chosen contours. Therefore, the method is in principle unacceptable and the obtained result with the method can not be correct.

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